NCRM National Centre for Research Methods

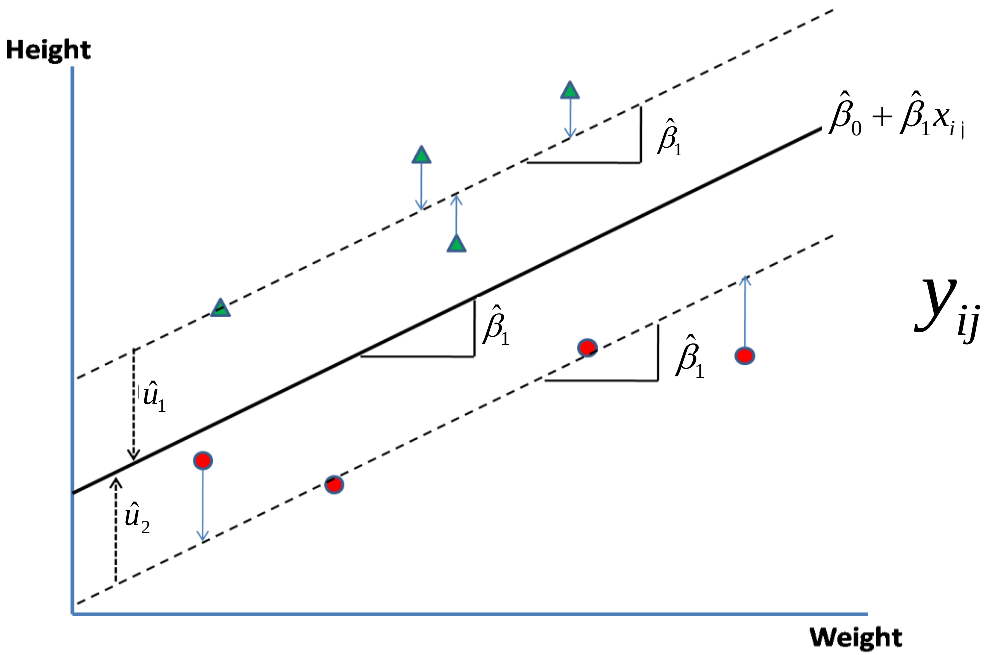
Multilevel Models

Session 3: Random coefficient models

Outline

- Random coefficient models
  - Allowing individual-level relationships to vary across groups
  - Linking individual and group level explanations – cross level interactions

Random intercept model of weight on height



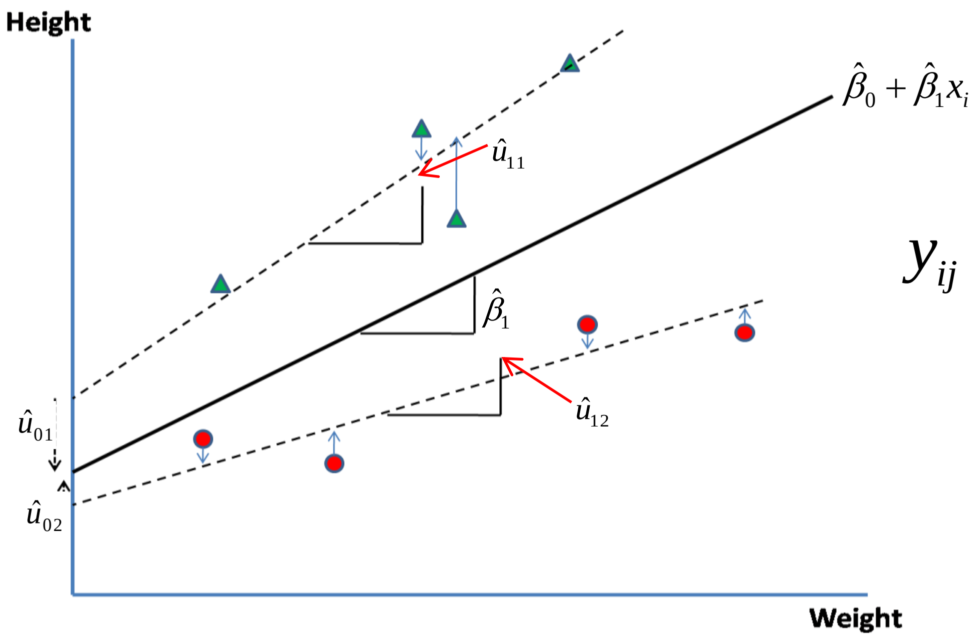
$$y_{ij} = \beta_0 + \beta_1 x_{ij} + u_j + e_{ij}$$

$$u_j \sim N(0, \sigma_u^2)$$

$$e_{ij} \sim N(0, \sigma_e^2)$$

- Previously we allowed average family height to be different in each family
- But assumed relationship with weight constant
- What if a unit increase in weight leads to **different** increases in height

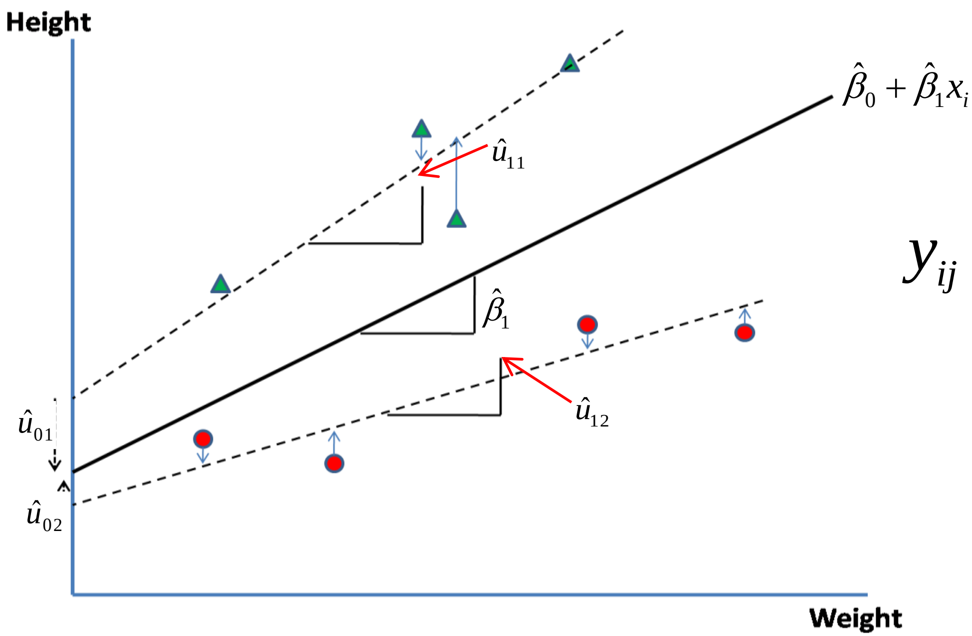
Random coefficient model of weight on height



$$y_{ij} = \beta_0 + \beta_1 x_{ij} + u_{0j} + u_{1j} x_{ij} + e_{ij}$$

- β_1 is average relationship between weight and height
- Slope of group j is $\beta_1 + u_{1j}$

Random coefficient model of weight on height



$$y_{ij} = \beta_0 + \beta_1 x_{ij} + u_{0j} + u_{1j} x_{ij} + e_{ij}$$

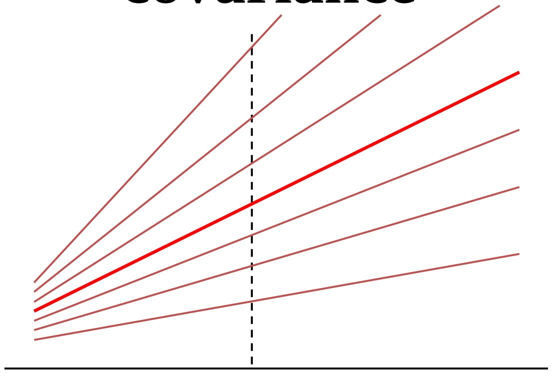
$$\begin{bmatrix} u_{0j} \\ u_{1j} \end{bmatrix} \sim N(0, \Omega_u) : \Omega_u = \begin{bmatrix} \sigma_{u0}^2 & \\ \sigma_{u01} & \sigma_{u1}^2 \end{bmatrix}$$

- Deviations from average intercept and coefficient assumed bivariate normal with variances σ_{u0}^2 σ_{u1}^2

- AND we also have information on how the residuals covary σ_{u01}

(+)ve covariance

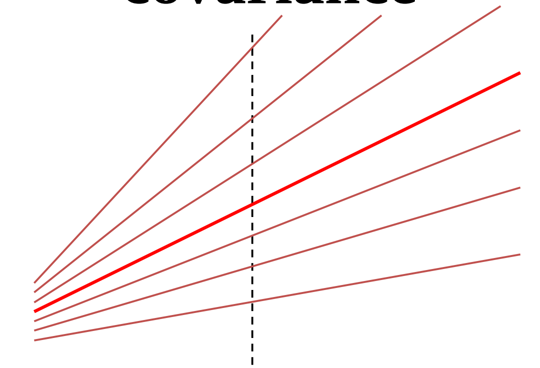
$Y = 2 + .5x$



Higher than average intercept = Higher (steeper) than average slope

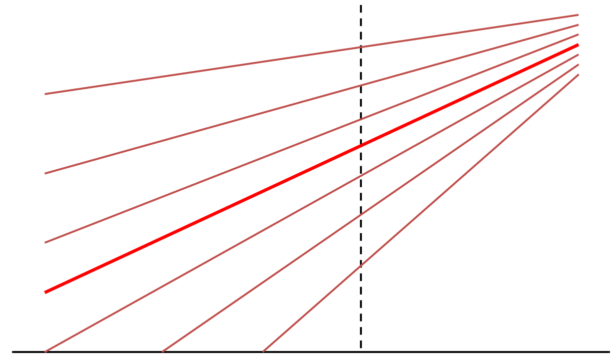
(+)ve covariance

$Y = 2 + .5x$



Higher than average intercept = Higher (steeper) than average slope

(-)ve covariance

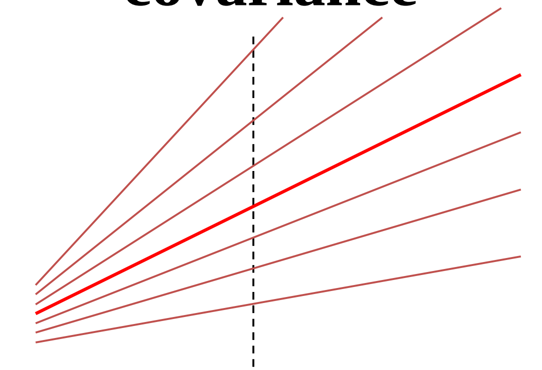


Higher than average intercept = Lower (flatter) than average slope

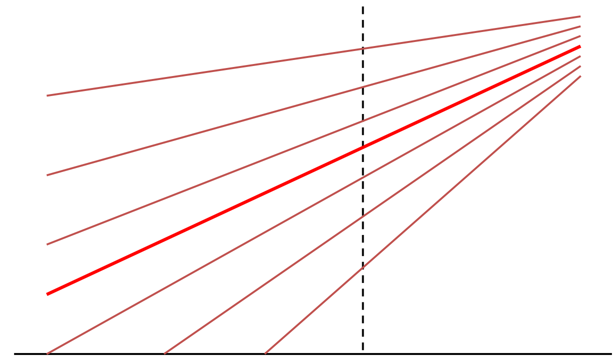
(+)ve covariance

(-)ve covariance

Y = 2 + .5x

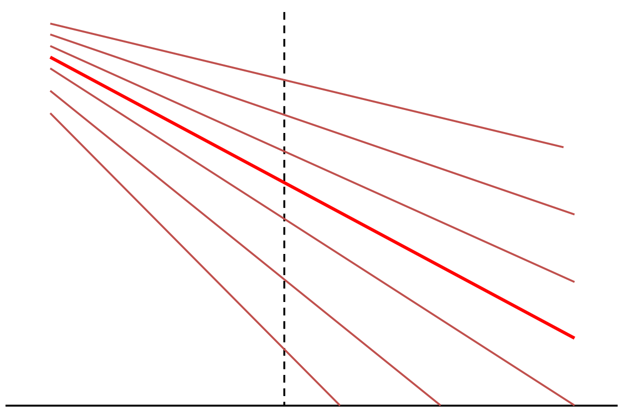


Higher than average intercept = Higher (steeper) than average slope



Higher than average intercept = Lower (flatter) than average slope

Y = 2 - .5x

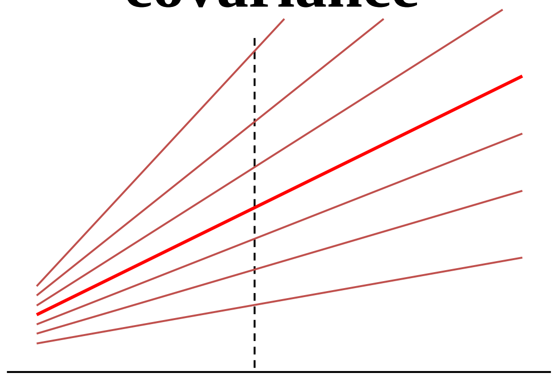


Higher than average intercept = Higher (flatter) than average slope

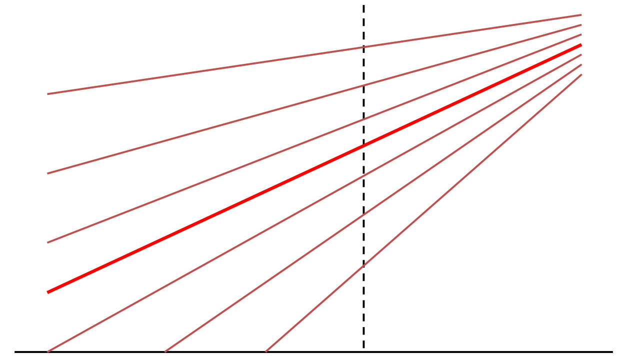
(+)ve covariance

(-)ve covariance

Y = 2 + .5x

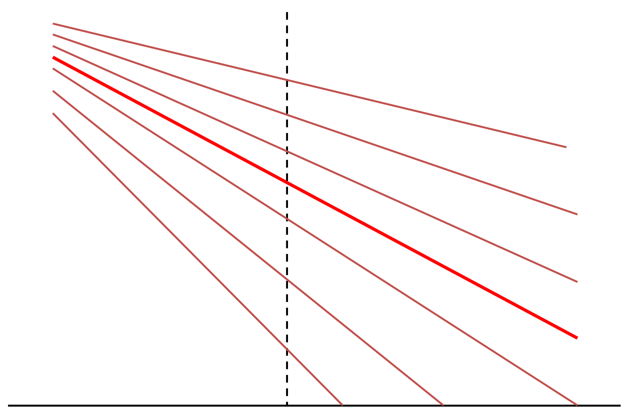


Higher than average intercept = Higher (steeper) than average slope

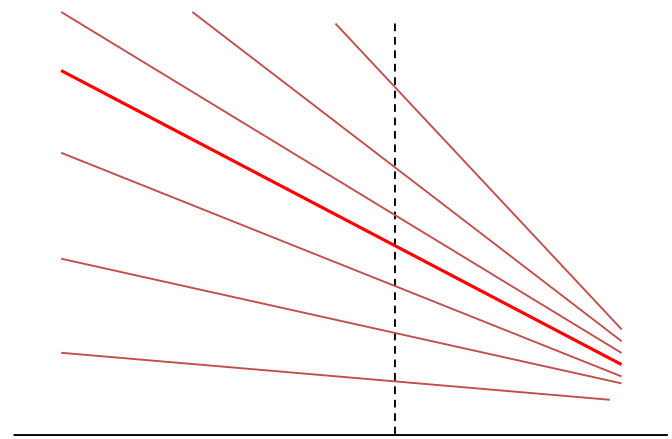


Higher than average intercept = Lower (flatter) than average slope

Y = 2 - .5x



Higher than average intercept = Higher (flatter) than average slope



Higher than average intercept = Lower (steeper) than average slope

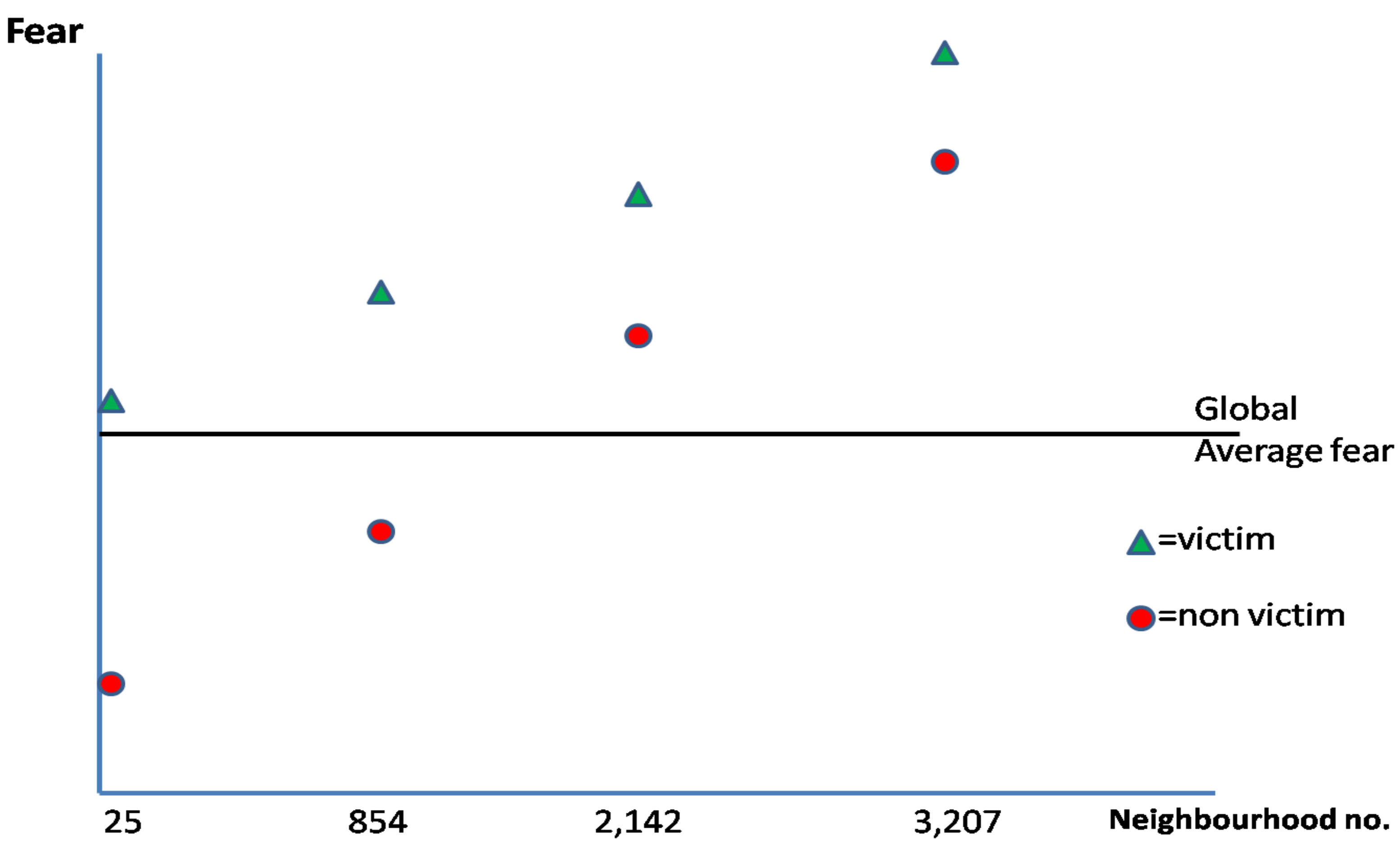
Example: Fear of Crime across neighbourhoods

RANDOM COEFFICIENT MODEL — Crime Survey for England and Wales, 2013/14

		MODEL 1	MODEL 2	MODEL 3
FIXED PART				
Intercept		0.027 (.009)	-.007 (.009)	-.007 (.009)
x_{1ij}	Age (in years)		-.004 (.001)	-.004 (.001)
x_{2ij}	Victim in last 12 months		.262 (.014)	.264 (.015)
x_{3j}	Crime Rate		.233 (.012)	.233 (.012)
RANDOM PART				
σ_e^2	Individual variance	0.863 (.008)	.855 (.008)	.853 (.008)
s_{u0}^2	Neighbourhood variance	0.145 (.007)	.104 (.006)	.097 (.007)
s_{u2}^2	Victim variance			.048 (.014)
s_{u02}	Covariance			-.022 (.008)

- Neighbourhood variance = how levels of fear vary across neighbourhoods for non-victims (e.g. when victim=0).
- Victim variance quantifies variation across neighbourhoods in the effect of being a victim

Graphical representation of random coefficient and covariance term



Cross level interactions

- Allow the effect of an explanatory variable on y to depends on the value of another (grouping) variable

- Do people experience context differently?
  - Place individuals directly within their context

$$y_{ij} = \beta_0 + \beta_1 x_{ij} + \beta_2 x_j + \beta_3 x_{1ij} x_{2j} + u_j + u_{1j} x_{ij} + e_{ij}$$

- Often included when individual level variables has a random coefficient

Example: Fear of Crime across neighbourhoods

CROSS LEVEL INTERACTION MODEL

Crime Survey for England and Wales, 2013/14

	MODEL 1	MODEL 2	MODEL 3	MODEL 4
FIXED PART				
Intercept	0.027 (.009)	-.007 (.009)	-.007 (.009)	-.011 (.009)
x_{1ij} Age (in years)		-.004 (.001)	-.004 (.001)	-.004 (.001)
x_{2ij} Victim in last 12 months		.262 (.014)	.264 (.015)	.268 (.015)
x_{3j} Crime Rate		.233 (.012)	.233 (.012)	.217 (.013)
$x_{2ij} * x_{3j}$ Victim * Crime Rate				-.067 (.021)
RANDOM PART				
σ_e^2 Individual variance	0.863 (.008)	.855 (.008)	.853 (.008)	.853 (.008)
s_{u0}^2 Neighbourhood variance	0.145 (.007)	.104 (.006)	.097 (.007)	.097 (.007)
s_{u2}^2 Victim variance			.048 (.014)	.046 (.014)
s_{u02} Covariance			-.022 (.008)	-.020 (.008)

- **Non-victim:** Fear = -.011 + .217 Crime Rate
- **Victim:** Fear = (-.011+.268) + (.217-.067) Crime rate

  Fear = .257 + .150 Crime Rate

Summary

- Random coefficient models can be used to more accurately account for differential associations between x and y across groups
- Cross-level interactions connect individual and group level explanations
- Extends readily to multiple random effects and additional levels
- Models also available for non-normal data (e.g. binary, categorical, ordered categories, poisson)

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